Confluence Techniques for Dependent Type Theory with Typed Conversion

Thiago Felicissimo & Théo Winterhalter

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INRIA

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(FKA Coq)

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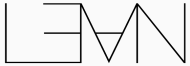

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This interaction is the most intricate feature of dependent types!

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Untyped Conversion Relation $t \equiv u$ on untyped terms (as in untyped λ-calculus)

$$\frac{}{(\lambda x.t)u \equiv t[u/x]} \qquad \frac{t \equiv u}{u \equiv t} \qquad \frac{t \equiv t'}{\lambda x.t \equiv \lambda x.t'} \qquad \dots$$

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Required for establishing correctness of implementations of type theory

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Typed conversion judgment $\Gamma \vdash t \equiv u : A$ defined mutually with typing $\Gamma \vdash t : A$

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Injectivity properties are usually shown with complex logical relation arguments

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- Lean's equality eliminator, with "*strong*" non-linear computation rule

$$\frac{\Gamma \vdash p : P[a/x] \qquad \Gamma \vdash e : a =_A a}{\Gamma \vdash \mathsf{transp}(x.P, p, a, a, e) \equiv p : P[a/x]}$$

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  Moreover, annotations help with semantics and can be elaborated away

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$$\frac{\Gamma \vdash A \equiv A_0 : \mathcal{U}^{\mathrm{n}} \qquad \Gamma \vdash t \Longrightarrow t' : \mathsf{Lift}^{\mathrm{n}}(A)}{\Gamma \vdash \mathsf{lift}_A(\mathsf{lower}_{A_0}(t)) \Longrightarrow t' : \mathsf{Lift}^{\mathrm{n}}(A)}$$

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Follows from two lemmas: (1) diamond property for $\Longrightarrow$; (2) inclusion of $\equiv$ in $(\Longrightarrow \cup \Longleftarrow)^*$

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Corollary Injectivity and non-confusion of type formers, eg

$$\Gamma \vdash \Pi(x : A).B \equiv \Pi(x : A').B' : \mathcal{U}^\ell \quad \rightsquigarrow \quad \Gamma \vdash A \equiv A' : \mathcal{U}^{\ell_A} \quad \wedge \quad \Gamma, x : A \vdash B \equiv B' : \mathcal{U}^{\ell_B}$$

$$\Gamma \vdash \Pi(x : A).B \equiv \mathsf{Nat} : \mathcal{U}^\ell \quad \rightsquigarrow \quad \bot$$

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See §4 and §5 of paper for more!

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Question: Can we go beyond η for Lift with these techniques?

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Many interesting features Proof irrelevance, η for Lift, non-linear rule for transp, etc

Application Correctness of conversion/type-checking, on usual non-annotated syntaxes

Limitations (see §7) No η for Π, Σ and Unit

Difference with the untyped case!

Takeaway Confluence is not at odds with typed conversion, but rather with η

Question: Can we go beyond η for Lift with these techniques?

Thank you for your attention!